TRANSFER PHENOMENA ACCOMPANYING OSCILLATIONS IN A TWO-COMPONENT MEDIUM

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For the example of the simplest two-component model, the mechanism of heat and mass transfer in a saturated porous medium when oscillations are imposed is considered.

1. Heat- and mass-transfer phenomena in two-component — for example, two-phase — media have been well studied for the case of constant or slowly varying velocity of the relative displacement of the components (filtration of liquid or gas in a porous solid). Further study of the heat and mass transfer with mutually oscillatory displacement of the components is of interest. A process of this kind in a homogeneous medium was considered in [1] for the theory of turbulence. Note that the model of the two-component medium considered below permits transition to a homogeneous medium in the presence of a velocity gradient in the latter.

The two-component medium is represented in the form of a system of alternating adjacent plates. One of each pair of neighboring plates is assumed to be motionless, while the second may move along the x axis, positioned at the interface, according to a periodic law. Between the plates, heat transfer occurs according to Newton's law. In the presence of a temperature gradient in the system along the x axis, the oscillations of the plates lead to heat transfer along the plates additional to the ordinary heat conduction, which is not considered here.

If the system is bounded in the direction of the x axis $(0 \leqslant x \leqslant l)$, and temperature T_0 is maintained at the ends, the mathematical formulation of the problem takes the form

$$mc_2\rho_2 \frac{\partial T_2}{\partial t} + mc_2\rho_2 u(t) \frac{\partial T_2}{\partial x} + \alpha (T_2 - T_1) = 0, \tag{1}$$

$$(1-m) c_1\rho_1 \frac{\partial T_1}{\partial t} - \alpha (T_2 - T_1) = 0, \tag{2}$$

$$T_2|_{x=0} = T_0, \quad T_2|_{x=l} = 0, \quad T_{1,2}|_{t=0} = 0, \quad t \geqslant 0. \tag{3}$$

The subscripts 1 and 2 refer to the motionless and oscillating components, respectively.

The system in Eqs. (1)-(3) is difficult to separate analytically; therefore, consideration is limited to one of the integrals of the given problem.

Dividing Eqs. (1) and (2) by $mc_2\rho_2$ and $(1-m)c_1\rho_1$, respectively, and subtracting the second from the first, the result is integrated with respect to x from 0 to l. The following simpler problem is obtained:

$$\frac{d\Theta}{dt} + \frac{\alpha}{\gamma}\Theta = -\frac{\alpha u(t)}{l}\int_0^l \frac{\partial T_2}{\partial x} dx = \frac{\alpha}{l} T_0 u(t),$$

$$\Theta|_{t=0} = 0.$$

Here

$$\Theta = \frac{\alpha}{l}\int_0^l (T_2 - T_1)\, dx; \quad \frac{1}{\gamma} = \frac{1}{mc_2\rho_2} + \frac{1}{(1-m) c_1\rho_1}.$$

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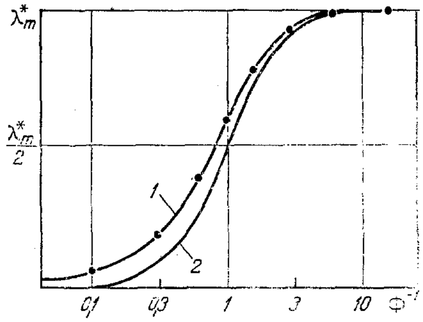

Fig. 1. Dependence of the effective heat-transfer coefficient on the parameter Φ^{-1} for discontinuous (1) and harmonic (2) oscillations.

The solution will take the form

$$\Theta = \frac{\alpha}{l} T_0 \int_0^t u(\tau) \exp\left[\frac{\alpha}{\gamma}(\tau - t)\right] d\tau. \tag{4}$$

The quantity Θ is the amount of heat exchanged between the components in unit volume and unit time.

2. Consider harmonic oscillations

$$u(t) = A\omega \sin \omega t, \quad X(t) = -A \cos \omega t + x_0.$$

Substitution into Eq. (4) leads to the following results:

$$\Theta = \frac{\alpha A T_0}{l} \frac{1}{\sqrt{1+\Phi^2}} \left[\sin(\omega t + \varphi) + \frac{\exp(-\omega \Phi t)}{\sqrt{1+\Phi^2}}\right],$$

where $\Phi = \alpha/\omega\gamma$, and the phase shift φ is determined by the expression

$$\sin\varphi = -\frac{1}{\sqrt{1+\Phi^2}}. \tag{5}$$

The last term in the solution obtained decreases over time, which offers the possibility of transforming to the following equation when $t \gg 1/\omega$, $t \gg \gamma/\alpha$:

$$\Theta_0 = \frac{\alpha A T_0}{l} \frac{1}{\sqrt{1+\Phi^2}} \sin(\omega t + \varphi).$$

The quantity Θ_0 characterizes the heat transfer between the components in steady thermal conditions, when the process of heat absorption by the medium is complete.

The heat-flux density along the x axis due to oscillations of the mobile component is

$$|q| = \langle \Theta_0 X \rangle = \frac{\alpha A^2 T_0}{2l\sqrt{1+\Phi^2}} |\sin\varphi|. \tag{6}$$

Thus, the heat transfer with oscillatory motion in a two-component medium depends on the phase shift between the oscillations of the mobile component and the heat flux between the components. It follows from Eqs. (5) and (6) that, at low frequencies ($\omega \to 0$, $\varphi \to 0$), this phase shift is $\pi/2$, and there is no heat transfer. With increase in frequency ($\omega \to \infty$, $|\varphi| \to \pi/2$), the phase shift tends to zero, and the heat flux along the x axis increases.

From Eqs. (5) and (6)

$$|\bar{q}| = \frac{\alpha A^2}{2} \frac{1}{1+\Phi^2} \frac{T_0}{l}.$$

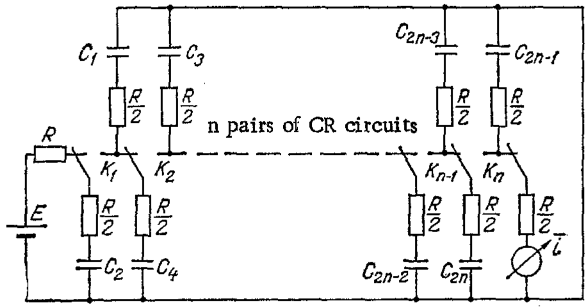

Fig. 2. Line diagram of electrical model.

The latter equation resembles the heat-conduction law written for a steady heat flux in a homogeneous medium with a temperature gradient T_0/l. Introducing the heat-transfer coefficient, which is an analog of the thermal conductivity

$$\lambda_1^* = \frac{\alpha A^2}{2}\frac{1}{1+\Phi^2}, \tag{7}$$

it follows from Eq. (7) that λ_1^* reaches a maximum when $\omega \gg \alpha/\gamma$: $\lambda_{1m}^* = (1/2)\alpha A^2$.

A limiting oscillation frequency $\omega_0 = \alpha/\gamma$ exists; at this frequency the heat-transfer coefficient is decreased by half. At small frequencies $\omega \ll \alpha/\gamma$, λ_1^* tends to zero. The limiting frequency increases with increase in heat-transfer coefficient between the components of the medium α and with decrease in the effective specific heat γ. For example, if $\alpha = 10^{10}$ $W/m^3 \cdot K$ [2], $\gamma^{-1} = 2\cdot 10^{-6}$ $m^3\cdot K/J$ are assumed for water-saturated sand, then $f_0 = \omega_0/2\pi = 3$ kHz.

3. Suppose that the mobile component undergoes a discontinuous change in position with respect to the immobile component, moving through a distance of 2A any number of times and remaining in each of the two states for an interval of $\tau/2$. Then

$$u(t) = \sum_{i=0}^{k} 2A(-1)^i \delta\left(t - i\frac{\tau}{2}\right), \quad k\frac{\tau}{2} < t < (k+1)\frac{\tau}{2}.$$

Substituting this expression into Eq. (4) and integrating gives

$$\Theta = \frac{2\alpha A T_0}{l}\exp\left(-\pi\Phi\frac{2t}{\tau}\right)\sum_{i=0}^{k}(-1)^i \exp(i\pi\Phi).$$

The result of calculating the sum is

$$\Theta = \frac{2\alpha A T_0}{l}\,\frac{1+(-1)^k \exp[(k+1)\pi\Phi]}{1+\exp(\pi\Phi)}\exp\left(-\pi\Phi\frac{2t}{\tau}\right).$$

The amount of heat transferred to unit volume of one component from the other in one half-period is now found

$$Q = \int_{k\tau/2}^{(k+1)\tau/2}\Theta(t)\,dt = \frac{\alpha A\tau}{\pi l\Phi}T_0\frac{1-\exp(-\pi\Phi)}{1+\exp(-\pi\Phi)}\{\exp[-\pi\Phi(k+1)] - (-1)^k\}.$$

On completion of the transient process $(t \gg \tau,\ t \gg \gamma/\alpha)$, the amount of heat circulating between the components is

$$Q_0 = \pm\frac{\alpha A\tau T_0}{\pi l\Phi}\,\frac{1-\exp(-\pi\Phi)}{1+\exp(-\pi\Phi)}.$$

The plus (minus) sign corresponds to the first (second) half-period of the oscillations. Taking into account that the heat stored by the moving component after the first half-period is transferred through a distance 2A and transmitted to the immobile component, the mean heat-flux density in the direction of the x axis may be found

$$|\bar{q}| = |Q_0|\frac{2A}{\tau} = \frac{2\alpha A^2}{\pi\Phi}\,\frac{1-\exp(-\pi\Phi)}{1+\exp(-\pi\Phi)}\,\frac{T_0}{l}.$$

The expression obtained for the heat-transfer coefficient is

$$\lambda_2^* = \frac{2\alpha A^2}{\pi\Phi}\,\frac{1-\exp(-\pi\Phi)}{1+\exp(-\pi\Phi)} = \frac{2\alpha A^2}{\pi\Phi}\,\mathrm{th}\,\frac{\pi\Phi}{2}. \tag{8}$$

Comparison of Eqs. (7) and (8), which is particularly clear in graphical form (Fig. 1), shows that the basic laws of the given transfer process are retained with significantly different types of oscillations. However, whereas the phase shift between the oscillatory motion and the oscillations of the heat flux between the components plays a considerable role in the harmonic oscillations, there is no phase shift in the discontinuous displacement.

A consequence of this is the absence of heat transfer as $\alpha \to \infty$ in the first case, whereas in the second from Eq. (8)

$$\lambda_2^*\Big|_{\alpha\to\infty} = \frac{4A^2\gamma}{\tau}.$$

4. An interesting feature of the second case is the possibility of experimental investigation on a simple electrical model consisting of a set of capacitors and resistors, switched, for example, by an electric relay. Replacing the thermal quantities by their thermal analogs with point parameters does not lead to any modeling error in the given case.

The electrical model (Fig. 2) has $n = l/2A$ pairs of elements, each of which consists of a capacitor and resistor in series, modeling the specific heat and heat-transfer coefficient, respectively. Switching of the elements is by switches K_1-K_n at the oscillation frequency of the system components. The electrical parameters of the model are related to the corresponding parameters of the system considered above by the following transformations:

$$\begin{gathered} C_1 \to (1-m)\,c_2\rho_2 2A, \quad C_2 \to mc_2\rho_2 2A, \\ T_0 \to E, \quad \bar{i} \to \bar{q}, \quad n \to l/2A, \quad R \to 1/2A\alpha. \end{gathered} \tag{9}$$

The mean value of the current drawn from the emf source in steady conditions, i.e., when the mean (over the period) charge at the capacitors is unchanged, is found. Obviously

$$\bar{i} = \frac{\Delta Q}{\tau} = \frac{\Delta U_1 C_1}{\tau} = \frac{\Delta U_2 C_2}{\tau}. \tag{10}$$

Here ΔQ and ΔU are the changes in charge and potential at the capacitors after the half-period $\tau/2$. The quantity ΔU_1 (or ΔU_2) may be found from the following systems of equations:

$$\begin{gathered} n\Delta U_1 + n\Delta U_2 + (2n+1)\,U = E, \\ \Delta U_1 + \Delta U_2 = (\Delta U_1 + \Delta U_2 + U)\,[1-\exp(-\varphi')], \\ \Delta U_1 C_1 = \Delta U_2 C_2, \end{gathered}$$

where U is the drop in potential at the resistance R at the end of capacitor recharging; and

$$\varphi' = \frac{\tau}{2}\,\frac{C_1+C_2}{RC_1C_2}. \tag{11}$$

Solving this system, ΔU_1 is found and substituted into Eq. (10), to give

$$\bar{i} = \frac{C_1C_2}{C_1+C_2}\,\frac{E}{(n+1)\,\tau}\left[\frac{2n+1}{n+1}\,(-\,e^{-\varphi'})^{-1} - 1\right]^{-1}.$$

Introducing the equivalent conduction of the circuit $\sigma = i/E$, and taking account of Eq. (11), it is found that

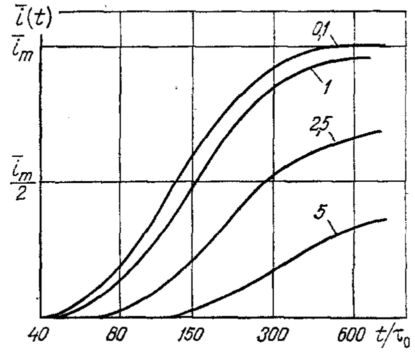

Fig. 3. Experimental curves of transient processes with the imposition of oscillations: τ_0 = RC; the figures on the curves give the ratio of the oscillation period to τ_0.

$$\sigma = \frac{1}{2(n+1)R\varphi'}\left[\frac{2n+1}{n+1}(1-e^{-\varphi'})^{-1}-1\right].$$

For the case when $n \gg 1$, the expression is somewhat simplified

$$\sigma = \frac{1}{2nR\varphi'}\,\frac{1-e^{-\varphi'}}{1+e^{-\varphi'}} = \frac{\operatorname{th}\frac{\varphi'}{2}}{2nR\varphi'}.$$

Using Eq. (9), an expression for the heat-transfer coefficient in the form in Eq. (8) is obtained.

To verify the theoretical relations and study transient conditions, a mock-up of the electrical model is built and measurements are made. The results of the measurements are shown by the points in Fig. 1. These results are obtained with R = 110 kΩ, $C_1 = C_2 = 1$ μF, n = 20. Electromagnetic relays are used as the switches. In Fig. 3, experimental curves of $\bar{i}(t)$ with the emf source switched on are shown; several curves corresponding to different oscillation frequencies are shown. The length of the transient processes is largely independent of the frequency in the high-frequency range and increases with decrease in frequency below the limiting value.

For further investigation of transfer processes with oscillations in a two-component medium, a full-scale model is constructed, consisting of a system of liquid-filled capillaries. Preliminary results of full-scale modeling qualitatively agree with the conclusions here outlined.

The given heat-transfer mechanism may play a significant role in the so-called thermoacoustic effect appearing as a considerable increase in effective heat conduction of a liquid-saturated porous medium under the action of acoustic oscillations [2]. Existing experimental data relating the appearance of this effect to the oscillation intensity and frequency agree with the above dependences. In addition, the marked appearance of the thermoacoustic effect in gas-saturated sand, which has perplexed investigators, may also be explained. In fact, within the framework of the given phenomenon, the basic difference between a gas and a liquid is that they have different specific heats. The specific heat has no influence on the heat-transfer coefficient at above-limiting frequencies. To a lesser extent than for a liquid, the heat-transfer coefficient of a gas may be compensated by the large oscillation amplitude of the fluid relative to the solid phase of the porous medium.

NOTATION

A, amplitude; C, capacity of the capacitor; c, specific heat; E, emf; f, frequency ($\omega = 2\pi f$); i, current; l, linear dimension of system; R, electrical resistance; T, temperature; t, time; U, potential; x, coordinate; α, specific heat-transfer coefficient; γ, effective specific heat; δ, Dirac delta function; λ*, heat-transfer coefficient; ρ, density; σ, equivalent conductivity of the circuit; τ, oscillation period; m, volume per fraction of the second component in unit volume of the medium.

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THEORY OF INTERFACIAL CONVECTION

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Using a simple thermocapillary convection problem as an example, we consider weakly nonlinear convective structures which arise near the surface of two liquids as a result of the Marangoni instability.

In the so-called Marangoni instability (a brief historical review is in [1]), near the surface between two immiscible liquids interesting stationary motion can develop. In some cases the motion is characterized by a high degree of order and there is a completely regular circulatory flow inside separate convection rolling cells which form a coherent structure (interfacial convection). In other cases the motion resembles random fluctuations in a turbulent fluid (interfacial turbulence). The different types of motion near the surface were observed in [2-6]; they can also be considered as dissipative structures developing in a nonequilibrium system [7, 8] or as a consequence of the randomizing behavior of dynamical systems with strange attractors [8, 9].

The presence of interfacial convection or turbulence leads to a significant increase in mass or heat transport across the surface (see [10, 11]) and is therefore of great interest. Physically, the Marangoni instability is most often due to the dependence of the surface tension on temperature or concentration of surface-active or inactive material (thermocapillary or concentration-capillary convection), but may also be caused by a dependence of the surface tension on the density of surface electric charges or dipoles, the polarization of the surface layer in an external electromagnetic field, the conformational structure of the surface layer, and so on (see [12]).

The theoretical studies in this field are almost entirely devoted to the linear analysis of the conditions for the onset of the Marangoni instability (representative examples can be found in [13-19]). Attempts to extend the analysis to nonlinear effects are rare [20-22], and except for numerical investigations, are limited to weakly nonlinear problems of interfacial convection. The method used in the present paper is essentially a variant of the small parameter method, applied previously to natural thermal convection in [23] and considered in detail in [24]. It is quite similar to the method of Lin [22, 25] and is based on the old classical works of Stuart and Watson [26, 27] on the nonlinear stability of plane Poiseuille and Couette flow. A similar method was applied to nonlinear instabilities and to the formation of space—time structures in thin liquid films deposited on substrates [28-30].

In essence, the method goes back to the well-known hypotheses of Landau [31] and Hopf [32] that the transition to turbulent motion can be thought of as a series of supercritical bifurcations of the set of periodic (or quasiperiodic) solutions of the Navier—Stokes equations describing the loss of stability in the analogous set of higher dimensionality, and on the possibility of stabilizing these solutions for not very large supercriticalities due to nonlinear interactions (the possibility of establishing regular periodic secondary flow under certain conditions). Although prior to the onset of natural turbulence, this hypothesis was shown to be untrue [33], it is correct in many other cases, in particular for interfacial convection.

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